Physics 11b
Lecture #8

Current and Resistance

S&J Chapter 27
First midterm this Thursday
- Covers up to and including capacitance
  - Lectures #1–#7, textbook chapters 23–26
- Five problems
  - Problem #1 is a series of multiple-choices
  - One of the problems is taken from the homework
- Bring a calculator
  - Make sure that the batteries are fresh
- No laptops, no PDAs, no cell phones

Bring a calculator
- Make sure that the batteries are fresh
Today’s Goals

- Define electric current $I$
  - Rate of electric charge flow
  - Also define electric current density $J$
- Introduce Ohm’s Law
  - Everyone knows already – right?
  - I’ll try to confuse you by talking about resistance, resistivity, conductivity and conductance
  - Ohm’s Law isn’t always correct – examples
- Introduce electric power
  - That’s the one measured in Watts
Current

- Current = movement of something continuous
  - Water “current” – in rivers, oceans, fish tanks
  - Air “current” = wind
- We often measure currents by the velocity
  - Think weather report: “Winds E at 20 to 30 mph” etc.
  - Not always the best way, though…

![Diagram of water flow through a pipe](image)

- Constant flow of water through a pipe may go faster or slower depending on the thickness of the pipe
  - What’s important in this case is \( \frac{\text{volume of water}}{\text{time}} \)
Define current: the amount of *something* that pass through *a given area* in unit time

- The “given area” is usually the entire cross section of a given conduit

\[
\text{Current} = V_1 A_1 = V_2 A_2
\]

- Current in the pipe is conserved as long as there is no leak

- The unit in this case is \((\text{m/s}) \times (\text{m}^2) = \text{m}^3/\text{s}\), or volume of water per second

- If we wanted the mass of water per second? → Multiply by the density of the water = 1 g/cm³ = 1000 kg/m³
Electric Current

- Electric charges are “flowing” in a conductor
  - Electric current = the amount of charge going through the cross section of the conductor
  
  \[ I = \frac{Q}{t} \]

  In case the flow is changing with time, consider very short period \( dt \)

  \[ I = \frac{dQ}{dt} \]

- Unit: Ampere = \( \frac{\text{Coulomb}}{\text{second}} \)

- It’s often useful to define current density

  \[ J = \frac{\text{Current } I}{\text{Area } A} \]

  Unit is A/m²
In most metals, free electrons move around and cause electric current. They are the current carriers.

Since electrons are negatively charged, the direction of the current is opposite to the direction of the electron velocity. This is confusing – and it’s all Ben’s fault.

In general, current carriers don’t have to be electrons. Salty water conduct current with Na\(^+\) and Cl\(^-\) ions. In semiconductors, positive carriers called “holes” coexist with electrons.
Imagine carriers, each with \( +q \), are moving at \( v_d \) m/s

- How many of them goes through the cross section \( A \) each second?

Carrier density

\[ n v_d A \]

Drift speed

Current is therefore:

\[ I = q n v_d A \]

What if there are more than one type of carriers?

- Let’s consider two types, \( q_1 \) and \( q_2 \)

\[ I_1 = q_1 n_1 v_{d1} A \quad I_2 = q_2 n_2 v_{d2} A \]

\[ I = \sum_i q_i n_i v_{di} A = (q_1 n_1 v_{d1} + q_2 n_2 v_{d2}) A \]

also

\[ J = q n v_d \]
Drift Speed

- Just how fast do free electrons move?
- Consider copper wire of cross section 1 mm$^2$ carrying 1 A
  - Density of copper is 8.95 g/cm$^3$, molar mass 63.6 g/mol
  - Suppose there is one free electron per atom
    \[
    n = \frac{6 \times 10^{23}/\text{mol} \times 8.95 \text{ g/cm}^3}{63.5 \text{ g/mol}} = 8.5 \times 10^{22} /\text{cm}^3 = 8.5 \times 10^{28} /\text{m}^3
    \]
  - Each electron has $q = -1.6 \times 10^{-19} \text{ C}$
    \[
    v_d = \frac{I}{qnA} = \frac{1 \text{ C/s}}{-1.6 \times 10^{-19} \text{ C} \times 8.5 \times 10^{28} /\text{m}^3 \times 1 \times 10^{-6} \text{ m}^2} = 7.4 \times 10^{-5} \text{ m/s}
    \]
  - Not very fast – It’s about 10 inch/hour
  - “Free” electrons aren’t very free
    - They bump into road blocks every few nanometers

Read Section 27.3 for more
Charge carriers move because of the $E$ field

- Wait! Is there supposed to be no $E$ field in conductors?
- That was when there was no current – Now there is

Carrier receives force $F = qE$

- It would accelerate (as Newton says) like a falling apple
- It doesn’t because it keeps bumping into positive ions
- It reaches a constant velocity ($= \text{drift velocity } v_d$) quickly

Drift velocity $v_d$ is proportional to the $E$ field

- So is current density: $J = nqv_d \propto E$
Ohm’s Law

- Current density is proportional to the electric field
  \[ J = \sigma E \]
  - \( \sigma \) is a constant called conductivity of the conductor

- **Ohm’s Law**: for many materials, the conductivity \( \sigma \) is constant over a wide range of electric field strength
  - This is an empirical law, with exceptions
    - Semiconductors are non-ohmic
  - \( \sigma \) may depend on other factors than \( E \) field
    - \( \sigma \) often depends on temperature
    - For some material \( \sigma \) depends on pressure
Consider a length of conductive wire

- Potential: \( \Delta V = V_b - V_a = E \ell \)
- Current: \( I = JA = \sigma EA \)

\[ \Delta V = \frac{\ell}{\sigma A} I \equiv RI \]

This is the “familiar” version of Ohm’s Law
- It’s more of a corollary to the “real” Ohm’s Law
- \( R \) is resistance of this wire

- Unit: Ohm = \( \frac{\text{Volt}}{\text{Ampere}} \)  
  Symbol: \( \Omega \)
Confused?

- Four quantities related to resistance

<table>
<thead>
<tr>
<th>Macroscopic</th>
<th>Resistance</th>
<th>$R = \Delta V/I$</th>
<th>ohm (Ω)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Conductance</td>
<td>$G = I/\Delta V$</td>
<td>siemens (S)</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Microscopic</th>
<th>Resistivity</th>
<th>$\rho = E/J$</th>
<th>Ω·m</th>
</tr>
</thead>
<tbody>
<tr>
<td>Conductivity</td>
<td>$\sigma = J/E$</td>
<td>$1/(Ω·m)$</td>
<td></td>
</tr>
</tbody>
</table>

- Units of conductivity and resistivity come from

\[
\sigma = \frac{\ell [m]}{R [Ω] A [m^2]} \Rightarrow \frac{1}{Ω \cdot m}
\]

- Macro and micro versions are connected by integrals
- Let’s look at a tricky example
A material of resistivity $\rho$ is filled between two highly-conductive tubes of radii $a$ and $b$ ($a < b$), length $\ell$.

- Potential difference $\Delta V$ between two tubes: $\Delta V = V_a - V_b$
- $E$ and $J$ are both pointing out.
- At radius $r$, $J(r) = \sigma E(r) = \frac{1}{\rho} E(r) \hat{r}$

Total current must be $J \times$ (area)

- $I = 2\pi r \ell J(r)$
- $J(r) = \frac{I}{2\pi r \ell}$
- $E(r) = \rho J(r) = \frac{\rho I}{2\pi r \ell}$

\[
\Delta V = -\int_b^a E(r) \, dr = \frac{\rho I}{2\pi \ell} \int_a^b \frac{dr}{r} = \frac{\rho I}{2\pi \ell} \ln \left( \frac{b}{a} \right)
\]

\[
R = \frac{\Delta V}{I} = \frac{\rho}{2\pi \ell} \ln \left( \frac{b}{a} \right)
\]
Resistivity and Temperature

- Resistivity of any material depends on temperature $T$
  - For metal, $\rho$ increases with $T$ as $\rho = \alpha T$
    - NB: $T$ is absolute temperature in Kelvin
    - $\alpha$ is temperature coefficient of resistivity
- Dependence may not be linear
  - Still, we can define temperature coefficient $\alpha$
    - $\alpha$ is a positive constant for metal
    - $\alpha$ is negative (and not constant) for semiconductors
  - Resistivity of semiconductors decrease with increasing temperature

\[
\alpha = \frac{d\rho}{dT}
\]
Superconductivity

- Resistivity of certain material becomes zero below a critical temperature $T_c$
  - $T_c$ typically very low – See Table 27.3 in textbook
  - Transition is abrupt
- Superconducting circuit can sustain current without potential difference $\Delta V$
  - If you have a loop of superconducting wires, and somehow started current circling around it, it will keep circling without power supply
  - Makes an excellent electromagnet
Connect a piece of resistive wire to a battery

- Every $t$ seconds, $Q = It$ Coulomb moves across potential $\Delta V$
- They lose energy $U = Q\Delta V$
- What happens to the energy?

Current $I$ flowing downstream across $\Delta V$ dissipates power $P = I\Delta V$ and turn it into heat

Unit: $A \cdot V = \text{Watt} = \text{Joule/second}$
Resistor $R$ is connected to a potential difference $\Delta V$.

Power dissipation is:

$$P = I\Delta V = I^2R = \frac{(\Delta V)^2}{R}$$

Simple example:

- A 60W light bulb dissipates 60W when connected to 120V
  - How much current does it draw?
  - What is the resistance?
- How about a 30W light bulb?
Not so simple example:

- Connect a 60W bulb and a 30W bulb in series
- Which one is brighter, and why?

Remember

\[ P = I \Delta V = I^2 R = \frac{(\Delta V)^2}{R} \]

- In a series connection, \( I \) is the same for both bulbs
- 60W bulb has smaller \( R \) than 30W bulb
- So…

We’ll do more of this in the next lecture
Summary

- **Electric current**: amount of charge that moves across a boundary surface in unit time
  - Current density \( J = \frac{I}{A} \)
  - For carrier density \( n \), charge \( q \), drift speed \( v_d \)
    \[
    J = n q v_d \quad I = n q v_d A
    \]

- **Ohm’s Law**: current density is proportional to \( E \) field
  \[
  J = \sigma E \quad \text{Integrate} \quad \Delta V = RI
  \]
  - Empirical law – good for most metals

- **Power dissipation** on a resistor
  \[
  P = I \Delta V = I^2 R = \frac{(\Delta V)^2}{R}
  \]