Physics 11b
Lecture #9

Direct Current Circuits

S&J Chapter 28
Midterm exam #1 has been graded
- Average 130 out of 150 points – Well done!
Electric current: amount of charge that moves across a boundary surface in unit time
- Current density $J = I / A$
- For carrier density $n$, charge $q$, drift speed $v_d$
  \[ J = nq v_d \quad I = nq v_d A \]

Ohm’s Law: current density is proportional to $E$ field
- Empirical law – good for most metals

Power dissipation on a resistor
- $P = I \Delta V = I^2 R = \frac{(\Delta V)^2}{R}$
Today’s Goals

- Discuss direct current (DC) circuits
  - Combination of resistors, capacitors and emfs
    - Electromotive force = what any battery has
- Resistor arithmetic
  - Many of you already know – Let’s remind ourselves
- Kirchhoff’s rules
  - Simple and straightforward rules that help you to solve DC circuit problems systematically
- RC circuit
  - One resistor and one capacitor and one battery
Batteries supply “electricity”
- Connect it to a resistor and we’ll find $\Delta V$ and $I$
- Connect it to a capacitor and we’ll find $\Delta V$ and $Q$

Looks like a battery is a $\Delta V$ generator
- We call a battery’s ability to produce a potential difference an electromotive force, or emf
- Unit of emf = Volt

Obvious enough
What is an EMF

- Inside a battery, chemical reactions move the charges from one contact to another
  - It’s like a pump pushing water up

Battery converts chemical energy to potential energy of charges

Pump converts kinetic energy to potential energy of water
Internal Resistance

- Connect a battery with an emf $\mathcal{E}$ to a resistor $R$.
  - The current should be $I = \frac{\Delta V}{R} = \frac{\mathcal{E}}{R}$.
  - True only for an ideal battery.
- Real batteries have internal resistance.

\[ \Delta V = \mathcal{E} - Ir < \mathcal{E} \]

Loss due to internal resistance.
Measuring $\mathcal{E}$ and $r$

- Internal loss, $I \cdot r$, is proportional to the current
  - We can measure $\Delta V$ with no current and get $\mathcal{E}$
  - This is open-circuit voltage
  - We can measure $\Delta V$ with current and find out $r$

Battery testers do this to determine if $r$ is too large
- “Dead” batteries have the same emf $\mathcal{E}$ as the fresh ones, but larger internal resistance $r$

\[ I = \frac{\mathcal{E}}{R + r} \]

\[ \Delta V = \mathcal{E} - Ir = \mathcal{E} \left(1 + \frac{r}{R + r}\right) \]
Resistor Arithmetic

- Let’s consider combinations of resistors

  ![Series diagram](image1)
  ![Parallel diagram](image2)

- Most of you already know how to do this
- If you don’t it’s easy enough to learn
Series Combination

- Connect two resistors $R_1$ and $R_2$ in series
  - Current $I$ through $R_1$ and $R_2$ are the same
  - Potential differences $\Delta V_1$ and $\Delta V_2$ are
    \[ \Delta V_1 = IR_1 \quad \Delta V_2 = IR_2 \]
  - Total potential $\Delta V$ must be sum of $\Delta V_1$ and $\Delta V_2$
    \[ \Delta V = \Delta V_1 + \Delta V_2 = I(R_1 + R_2) \]

- Series Resistor Rule:
  \[ R_{\text{equiv.}} = R_1 + R_2 \]
Parallel Combination

- Connect two capacitors $R_1$ and $R_2$ in parallel
  - Potential $\Delta V$ across $R_1$ and $R_2$ are the same
  - Current through the resistors are
    \[ I_1 = \frac{\Delta V}{R_1}, \quad I_2 = \frac{\Delta V}{R_2} \]
  - Combined, they draw the total current
    \[ I = I_1 + I_2 = \Delta V \left( \frac{1}{R_1} + \frac{1}{R_2} \right) \]

- Parallel Resistor Rule:
  \[ \frac{1}{R_{\text{equiv.}}} = \frac{1}{R_1} + \frac{1}{R_2} \]
Resistor Arithmetic

- For arbitrary number of resistors

\[ R_{\text{equiv.}} = R_1 + R_2 + R_3 + \cdots + R_n \]

\[
\frac{1}{R_{\text{equiv.}}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \cdots + \frac{1}{R_n}
\]

- Ex:

\[
\begin{align*}
20\Omega & \quad 30\Omega \\
50\Omega && 50\Omega && 50\Omega
\end{align*}
\]

\[
\begin{align*}
50\Omega \\
50\Omega
\end{align*}
\]

\[
25\Omega
\]
Kirchhoff’s Rules

- At any point in a circuit, the sum of the incoming current equals to the sum of the outgoing current

\[ \sum I_{\text{in}} = \sum I_{\text{out}} \]  

Junction Rule

- Around any loop in a circuit, the sum of the potential differences is zero

\[ \sum \Delta V = 0 \]  

Loop Rule

- These are restatements of what we know perfectly well
  - Electric charge is conserved → What comes in must go out
  - Electric potential is conservative → Going around a loop must take you back to the original potential
Using Kirchhoff

- Let’s do a simple example: Find the current through $R_3$
- **Step 1:** Define currents
  - One for each “branch” of the circuit
  - Direction doesn’t matter – Pick one
- **Step 2:** Apply the junction rule
  - Junction $A \rightarrow I_1 + I_2 = I_3$
  - Junction $B \rightarrow I_3 = I_1 + I_2$
- **Step 3:** Apply the loop rule
  - Top loop $\rightarrow \mathcal{E}_1 - I_1R_1 - I_3R_3 = 0$
  - Bottom loop $\rightarrow -\mathcal{E}_2 - I_2R_2 - I_3R_3 = 0$

What’s this minus sign doing here?
To apply the loop rule, you must decide which direction to go around each loop.

Either will work, just pick one.

As you go around the loop,

An emf counts as $+\mathcal{E}$ if you pass through it from $-$ to $+$.

A resistor counts as $-IR$ if the current is flowing the direction you are going.

If you are going this way …

$$-\mathcal{E}_2 - I_2R_2 - I_3R_3 = 0$$
Back to the Problem

- We’ve found so far
  - Junctions: \[ I_1 + I_2 = I_3 \]
  - Top loop: \[ \epsilon_1 - I_1 R_1 - I_3 R_3 = 0 \]
  - Bottom loop: \[ -\epsilon_2 - I_2 R_2 - I_3 R_3 = 0 \]

- We introduced 3 unknown currents and found 3 equations
  - Solving them together

\[ I_1 = \frac{(R_2 + R_3)\epsilon_1 + R_3\epsilon_2}{R_1R_2 + R_1R_3 + R_2R_3} \]
\[ I_2 = \frac{R_3\epsilon_1 + (R_1 + R_3)\epsilon_2}{R_1R_2 + R_1R_3 + R_2R_3} \]
\[ I_3 = \frac{R_2\epsilon_1 - R_1\epsilon_2}{R_1R_2 + R_1R_3 + R_2R_3} \]
Kirchhoff Strategy

- **Step 1: Define currents**
  - One for each “branch” of the circuit
  - Pick the direction of the current

- **Step 2: Apply the junction rule**

- **Step 3: Apply the loop rule**
  - Pick the direction of the loop
  - Pay attention to the polarity

- You’ll have just the right number of equations to solve for all the currents you defined in Step 1
  - The rest is straightforward (it may be tedious)
Capacitors in DC Circuits

- Two plates of a capacitor are insulated from each other
  - Current cannot flow through it
- In a DC circuit, where the current is constant, capacitors don’t do anything – as if it’s not there
- The story is different if the current is not constant
  - Alternate Current (AC) circuits often use capacitors
  - When a switch is thrown (opened or closed) in a DC circuit, the current must change momentarily before settling into a new constant value → Capacitors do something in the process
  - Let’s look into this transient process
Simple RC Circuit

- With the switch open, the current $I$ must be zero
  - Assume $Q = 0$ as well
- Close the switch
  - Initially, $C$ has zero charge, so the potential difference around $C$ is 0
  - Kirchhoff’s loop rule says
    \[ \mathcal{E} + 0 - IR = 0 \]
    \[ I = \frac{\mathcal{E}}{R} \]
- The current brings electric charge into the capacitor
  \[ \frac{dQ}{dt} = I \]
  Capacitor $C$ is being charged up by the current $I$
- As $Q$ increases, potential difference appears around $C$
Simple RC Circuit

- Time $t$ after closing the switch
  - $C$ holds $Q = Q(t)$
  - Kirchhoff’s loop rule
    \[ \mathcal{E} - \frac{Q(t)}{C} - I(t)R = 0 \]
  - Combine with
    \[ \frac{dQ(t)}{dt} = I(t) \]
    \[ \frac{dI(t)}{dt} = -\frac{I(t)}{RC} \]
  - Solution to this differential equation is
    \[ I(t) = I_0 e^{-\frac{t}{RC}} \]
    \[ Q(t) = -RCI_0 e^{-\frac{t}{RC}} + \text{const.} \]
Simple RC Circuit

\[ I(t) = I_0 e^{\frac{t}{RC}} \]

\[ Q(t) = -RCI_0 e^{\frac{t}{RC}} + \text{const.} \]

- Initial conditions at \( t = 0 \)
  \[ I(0) = I_0 = \frac{\mathcal{E}}{R} \]
  \[ Q(0) = 0 \]

- Finally
  \[ I(t) = \frac{\mathcal{E}}{R} \exp\left(-\frac{t}{RC}\right) \]
  \[ Q(t) = \mathcal{E}C\left(1 - \exp\left(-\frac{t}{RC}\right)\right) \]
Simple $RC$ circuits “relax” toward the stable equilibrium exponentially

- Time-dependence of the current and charge has a $\exp(-t/RC)$ form
- Product $RC$ has the dimension of time (really?)
- It’s called the time-constant of the circuit

Real-world circuits contain Rs and Cs everywhere

- How fast the circuits can perform their functions often determined by the $RC$ time constant
- $RC$ circuits are also used in timers, frequency filters, power regulators, etc.
- We will do more of it with the AC circuits
Summary

- **Electromotive forces (emfs)**
  - Batteries are made of an emf and an internal resistance

- **Resistor arithmetic**
  \[
  R_{\text{series}} = R_1 + R_2 + R_3 + \cdots + R_n
  \]
  \[
  \frac{1}{R_{\text{parallel}}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \cdots + \frac{1}{R_n}
  \]
  - Opposite to capacitors!

- **Kirchhoff’s rules**
  \[
  \sum I_{\text{in}} = \sum I_{\text{out}} \quad \text{Junction Rule}
  \]
  \[
  \sum_{\text{loop}} \Delta V = 0 \quad \text{Loop Rule}
  \]
  - Loop rule requires attention to the polarity

- **RC circuits**
  - Time-dependence as \[ \exp\left(-\frac{t}{RC}\right) \]  
  - Time constant