Physics 11b
Lecture #24

Quantum Mechanics
Theory of special relativity is based on two postulates:
- Laws of physics is the same in all reference frames
- Speed of light is the same in all reference frames

There is no absolute time
- Time dilation: $\Delta t' = \frac{\Delta t}{\sqrt{1 - \frac{v^2}{c^2}}}$
- Length contraction: $L' = L \sqrt{1 - \frac{v^2}{c^2}}$
- Galilean transf. $\rightarrow$ Lorentz transf.
- Addition of velocities $u = \frac{u' + v}{1 + \frac{uu'}{c^2}}$

\[
\begin{cases}
  x' = \gamma(x - vt) \\
  y' = y \\
  z' = z \\
  t' = \gamma(t - \frac{v}{c^2} x)
\end{cases}
\]
Introduction to Quantum Mechanics
- Particle nature of light $\rightarrow$ Photon
  - Why do we think light is made of photons?
- How can light be waves and particles at the same time?
- How does this theory extend to “normal” particles?

Mainly historical background for the development of Quantum Mechanics
- I hope you’ll have a chance to learn QM in depth later
- Because it’s a true paradigm shift that inspired the words “paradigm shift”
  - Read Thomas Kuhn
- Because it’s cool
Photoelectric Effect

- When light hits a metal surface, electrons come out
  - It’s called **photoelectric effect**
- Taking an electron out of metal requires energy $W$
  - Light must be supplying this energy
  - You need certain intensity to get any electron out
  - Energy $E_e$ of the electron that came out should be larger for higher light intensity

$$E_e + W \propto I$$

- Problem: experiments don’t agree with our theory
Experiment vs. Theory

- Experiments have shown
  - Energy $E_e$ of electrons is independent of light intensity $I$
    - Expected to increase with $I$
  - At low $I$, fewer electrons come out, but some do come out
    - Expected no electrons below a certain intensity
- Totally unexpected fact: $E_e$ depends on the frequency $\nu$ of the light as
  $$E_e + W = h\nu$$
  - Planck’s constant $h = 6.626 \times 10^{-34} \text{ J} \cdot \text{sec}$
  - Einstein’s explanation: smallest unit of light (photons) carries an energy $E = h\nu$
Photon as a Reality

- R.A. Millikan’s careful experiment confirmed the correctness of Einstein’s theory beyond doubt
  - Existence of photon was clearly established
  - Einstein (1922) and Millikan (1923) won Nobel
- But now we are back to the original particle-vs.-wave argument
  - Young’s experiment and countless other evidences support that light is made of waves
  - Millikan considered the photon theory “quite unthinkable” because of it’s inconsistency with the interference and diffraction phenomena
You have probably heard/read that “light behaves sometimes like particles and sometimes like waves”

Is that a clear statement, or what?

We must know better than “sometimes” or “like”

Photoelectric effect gives us a starting point

Absorption of light by matter (electrons) occur with the minimum unit of energy $E = h\nu$

Generalize this into a hypothesis

Light is emitted and absorbed as particles (photons), each particle carrying energy $E = h\nu$
Energy, Momentum, Intensity

- We know for any waves \( \frac{\text{Energy}}{\text{Velocity}} = \text{Momentum} \)
  - Velocity = \( c \) for light
  - If each photon has \( E = h\nu \), then it should have
  - Using \( \hbar \equiv h/2\pi \), we can say \( E = \hbar\omega \) \( p = \hbar k \)

- Intensity of light is determined by the number of photons emitted/received per unit time

- Example: a laser pointer with \( P = 5 \text{ mW} \) and \( \lambda = 650 \text{ nm} \)
  \[
  \frac{P}{h\nu} = \frac{P\lambda}{hc} = \frac{5 \times 10^{-3} \times 650 \times 10^{-9}}{6.6 \times 10^{-34} \times 3 \times 10^8} = 1.6 \times 10^{16} \text{ photons/sec}
  \]
  - That’s quite a few photons…
I said in Lecture #21 that Young’s experiment had demonstrated light’s wave-ness.

Let’s review it assuming that light is made of photons.

Number of photons arriving at screen is

$$\frac{I}{hv} = N_0 \cos^2 \frac{\pi dy}{\lambda D} \text{photons/sec} \cdot \text{m}^2$$
Young’s Experiment

- Darken the light until photons arrive one at a time

- Photons hit the screen at random locations, one by one
- Distribution still follows the interference
  - If you wait long enough, you get the same pattern
- Each single photon is affected by the interference
  - Interference determines the probability $\mathcal{P}(y)$ of each photon arriving at location $y$

$$\mathcal{P}(y) \propto \cos^2 \frac{\pi dy}{\lambda D}$$

But how?
As a particle, each photon can go through only one slit
- With one slit, however, interference does not occur

Each photon is affected by the presence of the both slits
- Conclusion: Each photon goes through both slits
  - OK with waves, but not with particles
    - Fractional photons have never been observed
- We are going in circles…
What Is a Particle, Anyway?

- Exactly what do we mean when we say something “is a particle”?
  - It is undivisible
  - It has a location \((x, y, z)\) at any time \(t\)

- The concept is an idealization/generalization from our experiences in observing ordinary objects
  - Watch a ball fly. Imagine it were infinitely small

- There are implicit assumptions in it
  - We must reexamine its validity
  - Keywords: “observing” and “watch”
Observing a Ball

- A ball flies as:
  \[
  \begin{cases}
  x(t) = v_{0x} t \\
  y(t) = v_{0y} t - \frac{1}{2} gt^2
  \end{cases}
  \]

- We watch the ball continuously
  \[\Rightarrow x(t), y(t) \text{ are continuous functions of time } t\]

- Watching = detecting the light scattered by the ball
  - Since light is made of photons, we know the location of the ball only when a photon hits it
  - Observation is not continuous

- We may relax the definition of “a particle” a little

A particle is found at a single location \((x, y, z)\) whenever it is observed
Photons

- From photoelectric effect, we assumed that light is emitted and absorbed as particles
  - We can detect emission/absorption of a photon by its energy
    - Photon source loses $E = h\nu$
    - Photon absorber gains $E = h\nu$
  - These processes occur at a particular point in space
- Between the two events, we cannot detect photons
  - To detect a photon, we must absorb it
- What a particle should do when nobody is watching?
  - More accurately: when no observation is possible even in principle?
Newtonian (continuous) view of motion is intuitive

- Ball does what it does whether or not anybody is watching

But we know it’s an approximation

- We use light for observation
- Light carries momentum $\rightarrow$ Pushes the ball
- Trajectory is affected (very slightly!) by the light

We can ignore this effect in the limit of weak light

- Photon theory breaks this approximation
  - Light cannot be made weaker than single photon

$$\begin{align*}
    x(t) &= v_{0x} t \\
    y(t) &= v_{0y} t - \frac{1}{2} gt^2
\end{align*}$$
Small Ball $\rightarrow$ Electron

- Let’s make the ball very small $\rightarrow$ An electron
  - Small target $\rightarrow$ Fewer photons hit it
  - Mass $m$ is small $\rightarrow$ Photon’s momentum is relatively large
  - Each hit by a photon changes the electron’s momentum $\rightarrow$ Electron staggers around
  - You don’t know its exact location until the next photon hits it

- Continuous picture is an approximation of this random-walk
  - Valid only when $p = mv$ of the object is much larger than the photon momentum $h/\lambda$
New "Particle" View

- Once we accept that light is made of photons, we find that we don’t know any better than:
  - A particle is found at a single location whenever it is hit by another particle

  - What happens between two interactions is unknown
  - Our experience and intuition offer no help

- Only way to know → Experiment
  - For light, we have enough experimental data that support wave-like propagation
  - New photon theory must satisfy them

We’ve pretty much painted ourselves into a corner
Photon Rules

- Light is made of photons that obey the following rules:
  - A photon is emitted or absorbed in unit of $E = h \nu$
    - Each process takes place at a specific point in space
  - Exact location at which a photon is found is random
    - i.e. it cannot be predicted even in principle
  - Probability of finding a photon at a given location can be calculated using the wave equation
    - This is the light intensity
- We reinterpret the EM wave equation as a probabilistic description of how photons travel from one place to another
Young’s Experiment

- Apply the rules to Young’s experiment

- An electron vibrates and emits a photon
  - It’s localized, i.e. position known accurately
    → Diffraction makes the direction of photon uncertain
    → Spherical waves

- EM waves travel through the slits and interfere

- We don’t know (nobody knows!) exactly where the photon will hit the screen

- But the probability can be calculated:
  \[ P(y) \propto \cos^2 \left( \frac{\pi dy}{\lambda D} \right) \]
Classical physics contains waves (light, sound, etc.) and particles that build up the objects.

We found light was in fact made of particles whose motion is described by waves.

Terribly awkward complication compared with the clean and intuitive Newtonian physics.

Plurality should not be posited without necessity.

Theory should be as simple as possible to explain things.

Can we restore some simplicity?

What if we assume that the motion of any particle is described by waves?
Try to describe the motion of a particle using waves

- Suppose \( E = \hbar \omega \) and \( p = \hbar k \) holds
- Kinetic energy of a particle with mass \( m \) is
  \[
  E = \frac{1}{2} m v^2 = \frac{p^2}{2m}
  \]
  \[
  \hbar \omega = \frac{\hbar^2 k^2}{2m}
  \]

- Waves that satisfy this relationship are solutions of
  \[
  i \hbar \frac{\partial}{\partial t} \Psi = - \frac{\hbar^2}{2m} \frac{\partial^2}{\partial x^2} \Psi
  \]

- Derivation requires complex calculus
- Wave function \( \Phi(x,t) \) describes the probability of finding the particle at position \( x \) and time \( t \)
Wave Packet

- Represent a particle with a wave packet
  - Waves that extend for a finite length
- What is the physical meaning of $\Psi$?
  - Analogous to $E(x, t)$ for photons
    - For photons, probability $P(x, t) \propto S \propto E^2$
  - $P(x, t) = |\Psi(x, t)|^2$
  - i.e. the square of the amplitude gives the probability of finding the particle at a particular point

- Speed of the wave packet is given by

$$v = \frac{\partial \omega}{\partial k} = \frac{\partial}{\partial k} \left( \frac{\hbar k^2}{2m} \right) = \frac{\hbar k}{m} = \frac{p}{m}$$

This is known as the group velocity

Agrees with the usual $p = mv$
De Broglie Wavelength

- Since particle = wave packet, it has a wavelength

- From \[ p = \hbar k \]

- \[ \lambda = \frac{2\pi}{k} = \frac{h}{p} \]

- It’s sizable only for very small momentum
  - Very light particles, e.g. electrons

- Even with electrons, De Broglie wavelengths are too short to observe any wave phenomena

  - That’s why we never suspected that particles could also be waves

  - We need to look at electrons moving in a very small space
    - How about a hydrogen atom?
Hydrogen Atom

- Electron is circling around a proton
  - Electrostatic force = centripetal force
    \[ \frac{q^2}{4\pi\varepsilon_0 r^2} = \frac{mv^2}{r} \]
    \[ p = mv = \sqrt{\frac{mq^2}{4\pi\varepsilon_0 r}} \]
    \[ \lambda = \frac{h}{p} = h\sqrt{\frac{4\pi\varepsilon_0 r}{mq^2}} \]

- Waves circle around the orbit
  - Must close the loop at full circle = \(2\pi r\)
    \[ 2\pi r = n\lambda \]
    \[ r = n^2 \frac{h^2\varepsilon_0}{\pi mq^2} \]
    \( n \) is any integer

- Energy of the electron is
  \[ E = \frac{1}{2} mv^2 = \frac{q^2}{8\pi\varepsilon_0 r} = \frac{1}{n^2} \frac{mq^4}{8\varepsilon_0 h^2} \]
Hydrogen Atom

- Radius $r$ is quantized to
  
  - Smallest radius is
    
    $$r_0 = \frac{h^2 \varepsilon_0}{\pi mq^2} = 5.3 \times 10^{-11} \text{m}$$

- This solves the problem of atom instability
  
  - Since there is a minimum orbit radius, the electron cannot lose all energy and fall into the proton

- It can still absorb/emit photons by moving between different “allowed” radii
  
  - We can calculate the wavelengths from
    
    $$E = \frac{1}{n^2} \frac{mq^4}{8 \varepsilon_0^2 h^2}$$
Hydrogen Lines

- Electron moves from orbit \( n \) to \( m \) and emit a photon
- Energy of the photon is
  \[ E = \frac{hc}{\lambda} = E_n - E_m = \frac{mq^4}{8\varepsilon_0^2h^2} \left( \frac{1}{n^2} - \frac{1}{m^2} \right) \]
- Wavelength satisfies
  \[ \frac{1}{\lambda} = \frac{mq^4}{8\varepsilon_0^2h^3c} \left( \frac{1}{n^2} - \frac{1}{m^2} \right) \equiv R_H \left( \frac{1}{n^2} - \frac{1}{m^2} \right) \]
- \( R_H \) is known as Rydberg constant
  \[ R_H = 1.097 \times 10^7 \text{ m}^{-1} \]
- Spectral lines of hydrogen matches this formula
  - Formula was found by Johann Balmer in 1885
  - Niels Bohr explained it with waves in 1913
- First sign of particle being waves
Quantum and classical mechanics are separated by just one logical leap: Position of an object is not necessarily defined when it is not observed

- Re-evaluation of what it means to be “a particle”
- Probabilistic interpretation of wave intensity

QM and Relativity are the two pillars of modern physics

- Combination of the two brought us quantum field theories, which explain almost everything in the physical world
- … except for gravity – Researches continue to pursue the Theory of Everything