Problem Set #1

- “Section” problems will be discussed at the sections.
- “Report” problems are due at noon on 9/25. They will be graded at 5 points each. Correct logic and clarity of explanation are required for a full score.
- Problems are taken from Goldstein. I typed them for those who have not purchased the book. I fixed obvious typos in the 3rd Edition of the book (the 2nd Edition had them correct).

Problem 1 (Section)
(Chapter 1, p. 33, Problem 19)
Obtain the Lagrange equations of motion for a spherical pendulum, i.e., a mass point suspended by a rigid weightless rod.

Problem 2 (Section)
(Chapter 1, p. 33, Problem 20)
A particle of mass $m$ moves in one dimension such that it has the Lagrangian

$$L = \frac{m^2 x^4}{12} + m x^2 V(x) - V^2(x), \quad (\text{ typo in the textbook!})$$

where $V$ is some differentiable function of $x$. Find the equation of motion for $x(t)$ and describe the physical nature of the system on the basis of this equation.

Problem 3 (Section)
(Chapter 1, p. 33, Problem 22)
Obtain the Lagrangian and equation of motion for the double pendulum illustrated in the figure on the right, where the lengths of the pendula are $l_1$ and $l_2$ with corresponding masses $m_1$ and $m_2$.

Problem 4 (Report)
(Chapter 1, p. 29, Problem 2)
Prove that the magnitude $R$ of the position vector for the center of mass from an arbitrary origin is given by the equation

$$M^2 R^2 = M \sum_i m_i r_i^2 - \frac{1}{2} \sum_{i,j} m_i m_j r_{ij}^2,$$

where $m_i$ and $r_i$ are the mass and the magnitude of the position vector of the $i$-th particle, $r_{ij}$ is the distance between the $i$-th and the $j$-th particles, and $M$ is the total mass of the system.

Problem 5 (Report)
(Chapter 1, p. 31, Problem 12)
The escape velocity of a particle on Earth is the minimum velocity required at Earth’s surface in order that the particle can escape from Earth’s gravitational field. Neglecting the resistance of the atmosphere, the system is conservative. From the conservation theorem for potential plus kinetic energy show that the escape velocity for Earth, ignoring
the presence of the Moon, is 11.2 km/s. (Yes, you need to look up a few constants. What are they?)
Bonus quiz (+2 points): Rockets are usually launched straight up. Does the angle of the velocity matter? Can you think of any reason to choose a particular direction?

Problem 6 (Report)
(Chapter 1, p. 31, Problem 13)
Rockets are propelled by the momentum reaction of the exhaust gases expelled from the tail. Since these gases arise from the reaction of the fuels carried in the rocket, the mass of the rocket is not constant, but decreases as the fuel is expended. Show that the equation of motion for a rocket projected vertically upward in a uniform gravitational field, neglecting atmospheric friction, is

\[ m \frac{dv}{dt} = -v' \frac{dm}{dt} - mg, \]

where \( m \) is the mass of the rocket and \( v' \) is the velocity of the escaping gases relative to the rocket. Integrate this equation to obtain \( v \) as a function of \( m \), assuming a constant time rate of loss of mass. Show, for a rocket starting initially from rest, with \( v' \) equal to 2.1 km/s ( typo in the textbook! ) and a mass loss per second equal to \( 1/60^{th} \) of the initial mass, that in order to reach the escape velocity the ratio of the weight of the fuel to the weight of the empty rocket must be almost 300!
Bonus quiz (+2 points): Using any computer software (Mathematica and Maple are probably the easiest), produce a graph of \( v \) as a function of \( t \).