Problem Set #3

- “Section” problems will be discussed at the sections.
- “Report” problems are due on 10/9 noon. Correct logic and clarity of explanation are required for a full score.
- Problems are taken from Goldstein with minor corrections.

Problem 1 (Section)
(Chapter 3, p. 128, Problem 11)
Two particles move about each other in circular orbits under the influence of gravitational forces, with a period $\tau$. Their motion is suddenly stopped at a given instant of time, and they are released and allowed to fall into each other. Prove that they collide after a time $\tau/4\sqrt{2}$.

Problem 2 (Section)
(Chapter 3, p. 128, Problem 15)
A meteor is observed strike Earth with a speed $v$, making an angle $\phi$ with the zenith. Suppose that far from Earth the meteor’s speed was $v'$ and it was proceeding in a direction making a zenith angle $\phi'$, the effect of Earth’s gravity being to pull it into a hyperbolic orbit intersecting Earth’s surface. Show how $v'$ and $\phi'$ can be determined from $v$ and $\phi$ in terms of known constants.

Problem 3 (Section)
(Chapter 3, p. 130, Problem 23)
Evaluate approximately the ratio of mass of the Sun to that of Earth, using only the lengths of the year and the lunar month (27.3 days), and the mean radii of Earth’s orbit ($8.149 \times 10^8$ km) and of the Moon’s orbit ($5.38 \times 10^5$ km).

Problem 4 (Report)
(Chapter 3, p. 128, Problem 10) 5 points
A planet of mass $M$ is in an orbit of eccentricity $e = 1 - \alpha$ where $\alpha \ll 1$, about the Sun. Assume that the motion of the Sun can be neglected and that only gravitational forces act. When the planet is at its greatest distance from the Sun, it is struck by a comet of mass $m$, where $m \ll M$, traveling in a tangential direction. Assuming that the collision is completely inelastic (that is, the planet and the comet becomes a single body after the collision), find the minimum kinetic energy the comet must have to change the new orbit to a parabola.
Problem 5 (Report)

(Chapter 3, p. 128, Problem 13) **8 points**

(a) Show that if a particle describes a circular orbit under the influence of an attractive central force directed toward a point on the circle, then the force varies as the inverse-fifth power of the distance.

(b) Show that for the orbit described the total energy of the particle is zero.

(c) Find the period of the motion.

(d) Find $\dot{x}$, $\dot{y}$, and $\dot{v}$ as a function of angle around the circle and show that all three quantities are infinite as the particle goes through the center of the force.

The origin of the force is on the circle—the left edge of the circle in the above figure. The particle goes right through it. Hint: express $u=1/r$ as a function of $\theta$, then write down the orbit equation

$$\frac{d^2u}{d\theta^2} + u + \frac{m}{l^2} \frac{dV}{du} = 0.$$  

Problem 6 (Report)

(Chapter 3, p. 129, Problem 17) **5 points**

One of the classic themes of science fiction is a twin planet (“Planet X”) to Earth that is identical in mass, energy, and momentum but is located on the orbit 180° out of phase with Earth so that it would be hidden by the Sun. However because of the elliptical nature of the orbit it would not always be completely hidden. Assume there is such a planet in the same Keplerian orbit as Earth in such a manner that it is in aphelion (furthest from the Sun) when Earth is in perihelion (closest to the Sun). Calculate to first order in eccentricity $e$ the maximum angular separation of the twin and the Sun as viewed from Earth. Could such a twin be visible from Earth? Suppose the twin planet were in an elliptical orbit having the same size and shape as that of Earth, but rotated 180° from the orbit of Earth, so that Earth and the twin would be in perihelion at the same time. Repeat your calculation and compare the visibility in the two situations.

Earth’s orbit has a semimajor axis of $1.50 \times 10^8$ km and the eccentricity of 0.0167. The radius of the Sun is $6.96 \times 10^5$ km.