Problem Set #5

- “Section” problems will be discussed at the sections.
- “Report” problems are due on 10/23 noon. Correct logic and clarity of explanation are required for a full score.
- Problems are taken from Goldstein with minor corrections.

Problem 1 (Section)
(Chapter 4, p. 182, Problem 20)
A sphere of radius \( a \) rolls on a horizontal plane without slipping. (Imagine a solid rubber ball on your desk.) Express this constraint in terms of the Euler angles. Show that the conditions are nonintegrable, and that the constraint is therefore nonholonomic.

Problem 2 (Section)
(Chapter 5, p. 236, Problem 21)
A compound pendulum consists of a rigid body in the shape of a lamina suspended in the vertical plane at a point other than the center of gravity. Compute the period of small oscillations in terms of the radius of gyration about the center of gravity and the separation of the point of suspension from the center of gravity. Show that if the pendulum has the same period for two points of suspension at unequal distances from the center of gravity, then the sum of these distances is equal to the length of the equivalent simple pendulum.

Problem 3 (Report)
(Chapter 4, p. 182, Problem 21) 5 points
A particle is thrown up vertically with initial speed \( v_0 \), reaches a maximum height and falls back to ground. Show that the Coriolis deflection when it again reaches the ground is opposite in direction, and four times greater in magnitude, than the Coriolis deflection when it is dropped at rest from the same maximum height.

Explain, using words and illustration, why this should be the case. You may consider this experiment being done at the equator for simplicity. (You should show that the direction is opposite and the magnitude is greater, but not the exact factor 4. Any of the several possible explanations is accepted as long as it is correct and is clearly presented.)

Problem 4 (Report)
(Chapter 5, p. 233, Problem 3) 5 points
Prove that for a general rigid body motion about a fixed point, the time variation of the kinetic energy \( T \) is given by

\[
\frac{dT}{dt} = \omega \cdot \mathbf{N}
\]
Problem 5 (Report)

5 points
It has been shown in the lecture that the angular velocity $\omega$ is expressed using the Euler angles by

$$
\omega = \begin{bmatrix}
\dot{\phi} \sin \psi \sin \theta + \dot{\theta} \cos \psi \\
\dot{\phi} \cos \psi \sin \theta - \dot{\theta} \sin \psi \\
\dot{\phi} \cos \theta + \dot{\psi}
\end{bmatrix}.
$$

Express the kinetic energy $T$ of a rotating rigid body, using the principal moments of inertia $I_1, I_2, I_3$, in terms of the Euler angles. Assuming there is no torque, write down Lagrange’s equation with respect to $\psi$ to obtain the third component of the Euler’s equation of motion.

Problem 6 (Report)

(Chapter 5, p. 234, Problem 7) 5 points
For the general asymmetrical rigid body, verify analytically the stability theorem by examining the solution of Euler’s equations for small deviations from rotation about each of the principal axes. The direction of $\omega$ is assumed to differ so slightly from a principal axis that the component of $\omega$ along the axis can be taken as a constant, while the product of components perpendicular to the axis can be neglected. Discuss the boundedness of the resultant motion for each of the three principal axes. (This is what I did on the blackboard to show the instability of rotation around the 2nd axis. Do it for all axes and confirm what I said with less handwaving and more logical explanation.)