Problem Set #6

- “Report” problems are due on 11/6 noon. Correct logic and clarity of explanation are required for a full score.
- Problems are taken from Goldstein with minor corrections.

Problem 1 (Section)

(Chapter 6, p. 274, Problem 13)
Two mass points of equal mass \( m \) are connected to each other and to fixed points by three equal springs of force constant \( k \), as shown in the diagram.

The natural length of each spring is \( a \). Each mass point has a positive charge \( +q \), and they repel each other according to the Coulomb law. Find the normal coordinates and their frequencies.
(Note: don’t forget that the springs are not at their natural lengths at the equilibrium.)

Problem 2 (Report)

(Chapter 6, p. 272, Problem 4) 8 points
Obtain the normal modes of vibration for the double pendulum, assuming equal lengths, but not equal masses, and small oscillation angles \( \theta_1 \) and \( \theta_2 \). Show that when the lower mass \( m_2 \) is small compared to the upper mass \( m_1 \), the two resonant frequencies are almost equal. If the pendula are set in motion by pulling the upper mass slightly away from the vertical and then releasing it, show that subsequent motion is such that at regular intervals one pendulum is at rest while the other has its maximum amplitude. This is the familiar phenomenon of “beats.”

Problem 3 (Report)

(Chapter 6, p. 274, Problem 14) 4 points
Find expressions for the eigenfrequencies of the electrical coupled circuit shown in the figure.

Hint: A capacitor \( C \) has a potential energy \( V = \frac{1}{2} Q^2 / C \) when charge \( Q \) is stored in it. An inductor \( L \) has a “kinetic” energy \( T = \frac{1}{2} LI^2 \) when current \( I \) is flowing through it. Charge conservation allows you to express the current as the time derivative of the charge. Also, the charge stored in the three capacitors must add up to zero. Once you have written down the total kinetic and potential energies, you can use the Lagrangian formalism just like you would on a mechanical system.
Problem 4 (Report)
(Chapter 6, p. 275, Problem 18) **8 points**
A particle of mass $m$ in an isotropic three-dimensional harmonic oscillator potential $V = \frac{1}{2}k(x^2 + y^2 + z^2)$ has a natural frequency of $\omega_0$. Assume that the particle has a charge $+q$, and that a uniform magnetic field $B$ is applied in the $+z$ direction. Find the normal modes and their frequencies. Discuss the results for the limits of strong and weak fields.
If the particle was placed at $x = a > 0$, $y = z = 0$ at $t = 0$ and released without initial velocity, how will the particle move with time? Describe the particle’s motion qualitatively assuming that the $B$ field is weak.