Problem Set #9

- “Report” problems are due on 12/11 noon. Correct logic and clarity of explanation are required for a full score.
- Problems are taken from Goldstein with minor corrections.

Problem 1 (Section)
(Chapter 8, p. 365, Problems 24 and 25)
(1) A uniform cylinder of radius $a$, length $l$, and density $\rho$ is mounted so as to rotate freely around a vertical axis. On the outside of the cylinder is a rigidly fixed spiral or helical track along which a mass point $m$ can slide without friction. Suppose the mass point start at the top of the cylinder, with both the mass and the cylinder at rest, and slides down under the influence of gravity. Using any set of coordinates, arrive at a Hamiltonian for the combined system of particle and cylinder, and solve for the motion of the system.

(2) Suppose that in the previous exercise the cylinder is constrained to rotate uniformly with angular frequency $\omega$. Set up the Hamiltonian for the particle in an inertial system of coordinates and also in a system fixed in the rotating cylinder. Identify the physical nature of the Hamiltonian in each case and indicate whether or not the Hamiltonians are conserved.

Problem 2 (Report)
(Chapter 8, p. 364, Problem 19)
The point of suspension of a simple pendulum of length $l$ and mass $m$ is constrained to move on a parabola $z = ax^2$ in the vertical plane. The pendulum is allowed to swing in the same $x$-$z$ plane. Derive a Hamiltonian governing the motion of the pendulum and its point of suspension. Obtain the Hamiltonian’s equations of motion. (It’s probably the easiest to use the $x$ coordinate of the suspension point and the angle of the pendulum to describe the system. The formulas still get quite ugly.)
**Problem 3 (Report)**

(Chapter 8, p. 365, Problem 23)

(a) A particle of mass \( m \) and electric charge \( e \) moves in a plane under the influence of a central force potential \( V(r) \) and a constant magnetic field \( \mathbf{B} \), perpendicular to the plane, generated by a static vector potential \( \mathbf{A} = \frac{1}{2} \mathbf{B} \times \mathbf{r} \).

Find the Hamiltonian using coordinates in the observer’s inertial system.

(b) Repeat part (a) using coordinates rotating relative to the previous coordinate system about an axis perpendicular to the plane with an angular rate of rotation

\[ \omega = -\frac{eB}{2m} \]  \( \text{Textbook is wrong.} \)

**Problem 4 (Report)**

(Chapter 8, p. 366, Problem 27)

(a) The Lagrangian for a system of one degree of freedom can be written as

\[ L = \frac{m}{2}(\dot{q}^2 \sin^2 \omega t + \dot{q}q\omega \sin 2\omega t + q^2 \omega^2). \]

What is the corresponding Hamiltonian? Is it conserved?

(b) Introduce a new coordinate defined by

\[ Q = q \sin \omega t. \]

Find the Lagrangian in terms of the new coordinate and the corresponding Hamiltonian. Is \( H \) conserved?