Teaching Staff

- Lecturer: Masahiro Morii
  - Tuesday/Thursday 11:30 – 1:00. Jefferson 256
- Section leaders: Srinivas Paruchuri and Abdol-Reza Mansourri
  - Two or three 1-hour sections per week
    - Date/time to be announced
    - Please fill out the student survey
- Course assistant: Carol Davis
  - She will have all course materials (problem sets, etc.)
Getting the Best Out of Us

- **Speak up!**
  - Ask questions **at any time** in the lectures and sections
  - Come to my **office hours**
    - Hours will be posted on the door/on the web

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<tr>
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<th>Morii</th>
<th>Paruchuri</th>
<th>Mansouri</th>
<th>Davis</th>
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<tbody>
<tr>
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- **Tell us when something in the class is not working for you**
Prerequisite Courses

- Physics 15a and 15b
  - Introductory Mechanics, Relativity and E&M
- Mathematics 21a and 21b (or equivalent)
  - Multivariate Calculus, Linear Algebra and Diff. Equations
- Take these prerequisites seriously
  - Without sufficient background, you’ll get lost quickly
  - If you *still* want to try, you need to get a written permission from the Head Tutors (Howard Georgi, David Morin)
Textbook

- *Classical Mechanics*, Goldstein, Poole and Safko
  - Required
  - Classic (literally) textbook. Originally published in 1950
    - A must-read for serious physicists
  - 3rd edition came out in 2001
    - 2nd edition still good (or *better*) – Get it if you can
- Will follow this textbook closely
  - Except for skipping a few advanced materials
    - It’s a 600-page book written for graduate students
Grading

- Grades will be based on a weighted average of:
  - Homework 40%
  - Mid-term exam 20%
    - 1-hour exam. After 10 lectures
  - Final exam 40%
    - Exam period. 3 hours
Homework

- **Problem sets** are distributed on Thursdays
  - Reports are due at the next week’s Thursday lecture
- **Typical format:**
  - 2–3 problems that will be discussed at sections
  - 3–4 problems you solve and turn in report
- **Work together in groups**
  - Groups will be assigned according to the Survey
  - Each of you must turn in your own report, though
Is This Course For Me?

- **Yes** if you have serious interest in Physics
  - **Definitely** if you want to try a career in Physics
- I’ll explain what we will study in the next 15 minutes
  - Plus a mini-lecture at the end
- If you are not sure at the end of this lecture
  - Come to my office (Lyman 239) and ask questions
  - I’ll be there everyday this week
What Is Classical Mechanics?
Mechanics concerns
- Motion of objects → Velocity and acceleration
- Cause of the motion → Force and energy

The objects move, but do not change their properties
- Idealized particles and rigid bodies
- Mass and moment of inertia are all what matters

Newton’s Three Laws of Motion
- You remember them, right?
- *Principia* (1687) pretty much wrapped it up
Classical vs. Modern

- “Modern” in physics means “20th century”
  - Quantum Mechanics
  - Relativity
- Classical Mechanics = pre-Quantum Mechanics
  - We include special relativity as well as E&M

- What happened between the 17th and 20th centuries?
Do We Care?

- We know Relativity and QM are the “right answers”
  - Newtonian Mechanics is a human-scale approximation
- Isn’t that enough?
  - Why should we learn the theory that has been superseded?

(An advanced course in classical mechanics) introduces no new physical concepts to the graduate student. It does not lead him directly into current physics research. Nor does it aid him, to any appreciable extent, in solving the practical mechanics problems he encounters in the laboratory.

*Goldstein, Preface to the First Edition*
Why Classical Mechanics?

- Three Good Reasons
  - Close connection to Modern Physics
    - Mastering CM gives you clearer view of QM
  - Powerful and versatile mathematical tools
    - Indispensable for advanced studies in physics
  - Reformulate familiar laws of physics using completely different approaches
    - Cleaner and more general formalism
    - It’s just cool
  - But first, let’s go back to the 17th century…
Newtonian Mechanics

- In principle, Newton’s Equation of Motion predicted the motion of any object(s) from the force
  - All you need is a big, fast computer
- In reality, life was not so easy
  - Intel was not founded until 1968
  - More fundamentally, the force may not be known
    - It may depend on time, location, or even velocity
    - E.g. ≥2 objects attracting each other by gravity
    - Solving 3-body problem turns out to be impossible

→ Quest for more powerful mathematics
Generalizing Equation of Motion

- Newtonian Mechanics deals with the object’s position
  - Goal: finding \( x = x(t), y = y(t), z = z(t) \)
  - 3 coordinates for each object \( \rightarrow \) \( 3N \) for \( N \) objects

- But there are infinite other ways to describe motion
  - E.g. a more natural way for a pendulum
    \[ x = L \cos \theta, \ y = -L \sin \theta, \ z = 0, \ \theta = \theta(t) \]
  - Number of free variables may not be \( 3N \)
  - Let’s call the new variables \textit{generalized coordinates}

- What are the Equations of Motion for generalized coordinates?
Lagrangian Formulation

- Newton’s Equation is about force \( \mathbf{F} = ma \)
  - You start from \( \mathbf{F} = \mathbf{F}(x, t) \) for all particles
    - \( 3N \) functions corresponding to \( 3N \) coordinates
- Forget the force. Introduce something else
  - Lagrangian: \( L = L(q, \dot{q}) \)
  - Lagrange’s Equation
    \[
    \frac{d}{dt} \left( \frac{\partial L}{\partial \dot{q}} \right) - \frac{\partial L}{\partial q} = 0
    \]
    Everything about this system is embodied in a scalar function \( L \)
- Lagrangian does not depend on a coordinate system
  - Switching to a different set of coordinates is a snap
Hamilton’s Principle

- **Hamilton’s Principle** derives Lagrange’s Equation from a simple rule:

  The time integral of $L$ is stationary for the path taken by an actual physical system

  $$\delta\int_1^2 L dt = 0$$

  Rather weird statement…

- **Newton’s Laws** were found by induction
  - “It is so because it agrees with many observations”
  - *Deriving* them from a principle means knowing *why it is so*
  - Not quite that dramatic, but it does suggest deeper reason
    - Eventually connected to Feynman’s path integral
  - Besides, calculus of variations is a useful technique
Hamiltonian Formulation

- Hamilton Equation
  - \((p, q)\) are canonical variables
  - \(H\) is a function called Hamiltonian

- Canonical variables ~ position and momentum
  - Aren’t they related by \(p = mv\)?

- Position and momentum as independent variables
  - Allows wider range of variable transformations than Lagrangian formulation
    - Formalism is clean, symmetric and cool
    - Eerie similarity to what QM does with the uncertainty principle

\[
\dot{q} = \frac{\partial H}{\partial p}, \quad \dot{p} = -\frac{\partial H}{\partial q}
\]
Success out of a Failure

- Quest for tools that solve the 3-body problem failed
  - Unless you count invention of computers
- Byproducts (Lagrangians and Hamiltonians) turned out to be the cornerstones of Quantum Mechanics
  - Development of QM was guided by analogies to Lagrangian and Hamiltonian formulations
    - Pioneers of QM grew up with Classical Mechanics
- Classical Mechanics is the missing link between Newton and Schrödinger
  - It allows you to fully appreciate QM
What We Will Study

- Lagrange’s Equations, Hamilton’s Principle
  - Central force problem
  - Rigid body motion
  - Oscillation
  - Extension to special relativity
- Hamilton Equation, Canonical transformations
  - Hamilton-Jacobi Equation
- Advanced stuff
  - Classical chaos?
  - Perturbation theory?
  - Field theory?
Lecture 1
Elementary Principles
(Goldstein Chapter 1)
Goals for Today

- Review basic principles of Newtonian Mechanics
  - Very quickly so that you don’t fall asleep
- Discuss motion of a single particle
  - Define standard notations and usages
  - Momenta, conservation laws, kinetic & potential energies
- You (should) already know all this
### Single Particle

- **Particle** = object with insignificant size
  - Electron in a CRT
  - Baseball thrown by a pitcher
  - Earth orbiting the Sun
- It has mass $m$, and location $\mathbf{r}$
  - Velocity $\mathbf{v} = \frac{d\mathbf{r}}{dt} = \dot{\mathbf{r}}$
  - Linear momentum $\mathbf{p} = m\mathbf{v} = m\dot{\mathbf{r}}$
- Newton’s 2nd law of motion $\mathbf{F} = \frac{d\mathbf{p}}{dt} = \dot{\mathbf{p}} = m\ddot{\mathbf{r}}$
Inertial System

1. The origin $O$ of $\mathbf{r}$ is somewhat arbitrary
   - A choice of origin $\rightarrow$ a reference frame
2. Inertial system $=$ a reference frame in which $\mathbf{F} = \dot{\mathbf{p}}$ holds
   - Newton’s 2$^{nd}$ Law should be stated more precisely as
     
     There exist reference frames in which the time derivative of the linear momentum equals to the force
   - And there are infinite number of such frames
Inertial Systems

- Consider two inertial systems $A$ and $B$
  - A particle is at $r_A$ in $A$, $r_B$ in $B$
  - Origin of $A$ is at $r_B - r_A$ in $B$

\[ F = m\ddot{r}_A = m\ddot{r}_B \rightarrow \ddot{r}_B - \ddot{r}_A = 0 \]
\[ \rightarrow \ddot{r}_B - \ddot{r}_A = \text{const} \]

Any two inertial systems are moving relative to each other at a constant velocity

- Equivalence of such systems was pointed out by Galileo
  - Hence the name Galilean system
Angular Momentum

- Define
  - Angular momentum \( \mathbf{L} = \mathbf{r} \times \mathbf{p} \)
  - Moment of force (torque) \( \mathbf{N} = \mathbf{r} \times \mathbf{F} \)

- From \( \mathbf{F} = \mathbf{\dot{p}} \) one can deduce \( \mathbf{N} = \dot{\mathbf{L}} \)
  - Too easy to write down here

- Subtlety: the definitions depend on the origin \( O \)
  - Because \( \mathbf{r} \) is defined from \( O \)
  - The equation \( \mathbf{N} = \dot{\mathbf{L}} \) holds for any origin

The order matters!
Momentum Conservation

- Two conservation theorems follow
  - From $F = \dot{p}$
    
    If the total force $F$ is zero, the linear momentum $p$ is conserved
  
  - From $N = \dot{L}$
    
    If the total torque $N$ is zero, the angular momentum $L$ is conserved

This is really getting too easy…
Work by External Force

- Particle moves from point 1 to 2 under force $\mathbf{F}$
  - Work $W_{12}$ done by the force $\mathbf{F}$ is defined by
    $$W_{12} = \int_1^2 \mathbf{F} \cdot ds$$

- One can define the kinetic energy $T \equiv \frac{mv^2}{2}$
  - Then derive $W_{12} = T_2 - T_1$

Work done equals to the change in the kinetic energy
If $W_{12}$ is the same for any possible path from 1 to 2, the force $\mathbf{F}$ is **conservative**

- $W_{12}$ depends only on the end points, not on the path

Equivalently, if you make a closed loop, the total work is zero

$$\int \mathbf{F} \cdot ds = \int_1^2 \mathbf{F} \cdot ds + \int_2^1 \mathbf{F} \cdot ds = 0$$
Potential Energy

- **F** is conservative $\iff$ **F** is expressed by $\mathbf{F} = -\nabla V(\mathbf{r})$
  - *V* is the **potential energy**

- **Work** $W_{12}$ is then expressed by $W_{12} = \int_{1}^{2} \mathbf{F} \cdot d\mathbf{s} = V_{1} - V_{2}$
  - Which was equal to $T_{2} - T_{1}$

\[ T_{1} + V_{1} = T_{2} + V_{2} \]

If the force is conservative, the total energy $T + V$ is conserved

**Energy Conservation Theorem**
Summary

- Reviewed basic principles of Newtonian Mechanics
  - Define standard notations and usages
  - Momenta, conservation laws, kinetic & potential energies
- I hope everything looked familiar, if boring
  - It will get better from here 😊
- Next: multi-particle system & constraints