1. (Purcell 10.23) Consider an oscillating electric field, \( E_0 \cos \omega t \), inside a dielectric medium that is not a perfect insulator. The medium has dielectric constant \( \epsilon \) and conductivity \( \sigma \). This could be the electric field in some leaky capacitor which is part of a resonant circuit, or it could be the electric field at a particular location in an electromagnetic wave.

(a) Show that the \( Q \) factor, as defined by Purcell equation 8.13

\[
Q = \frac{\omega}{\frac{\text{energy stored}}{\text{average power dissipated}}},
\]

is \( \epsilon \omega / 4\pi \sigma \) for this system.

(b) Evaluate \( Q \) for seawater at a frequency of 1 GHz. (The resistivity of seawater is \( \rho = 25 \, \Omega \cdot \text{cm} \). The dielectric constant of seawater may be taken to be the same as that of pure water at the same frequency, shown below.)

(c) What does your result suggest about the propagation of wavelength \( \lambda = 10 \, \text{cm} \) waves through seawater?

2. (Purcell 11.4) At the north magnetic pole the earth’s magnetic field is vertical and has a strength of 0.62 gauss. The earth’s field at the surface and further out is approximately that of a central dipole.

(a) What is the magnitude of the dipole moment in ergs/gauss?

(b) What is the magnitude of the dipole moment in joules/tesla?

(c) Imagine that the source of the field is a current ring on the “equator” of the earth’s metallic core, which has a radius of 3000 km, about half the earth’s radius. How large would that current have to be?
3. (Purcell 11.2) Previously, we calculated the field at a point on the axis of a circular current ring of radius $b$, carrying current $I$. The result is given by Purcell equation 6.41:

$$B_z = \frac{2\pi b^2 I}{c r^3} = \frac{2\pi b^2 I}{c(b^2 + z^2)^{3/2}}$$

(a) What is the magnetic dipole moment of this current ring?
(b) What is the contribution of this dipole moment to the field on the axis of the current ring?
(c) Note that despite being a perfectly symmetric current ring, this is not a perfect magnetic dipole. What makes a perfect dipole? (Please answer this specifically in terms of limits.)
(d) Show that for $z \gg b$ this current loop does approach the field of a magnetic dipole, and find how far out on the axis one has to go before the field has come within 1% of the field that a perfect dipole of the same dipole moment would produce at that point.

4. (Purcell 11.12) We want to find the potential energy of the two dipoles in the configuration shown below.

(a) Find an orientation of the two dipoles such that you can bring them from infinity to separation $r$ with no work.
(b) Calculate the work done to rotate $\mathbf{m}_1$ into its final orientation while holding $\mathbf{m}_2$ fixed.
(c) Calculate the work done to rotate $\mathbf{m}_2$ into its final position.

5. Use the results of the previous problem to compute the energies of each of the dipole pairs shown below.

(a) (b) (c) (d)

(e) What does this tell you about the forces involved in a ferromagnet?