1. Compute the curl and divergence of each of the following vector fields, and state which ones could represent $\mathbf{E}$ fields. For those that can be $\mathbf{E}$ fields, find the potential function $\varphi(x, y, z)$.

   (a) $F_x = x + y; F_y = -x + y; F_z = -2z$
   (b) $G_x = 2y; G_y = 2x + 3z; G_z = 3y$
   (c) $H_x = x^2 - z^2; H_y = 2; H_z = 2xz$

2. In the lecture we have seen $\nabla \times (\nabla \varphi) = 0$ for any scalar field $\varphi$. A similar-looking relation $\nabla \cdot (\nabla \times \mathbf{A}) = 0$ holds for any vector field $\mathbf{A}$. Let’s prove the latter in two ways:

   (a) Using the formula for $\nabla$ in Cartesian coordinates, work out the string of second partial derivatives that $\nabla \cdot (\nabla \times \mathbf{A})$ implies.
   (b) Consider the surface $S$ in the figure below. The closed curve $C$ almost cut $S$ into two halves. Invoke Stokes and Gauss with suitable arguments.

   ![Diagram of S and C](image)

3. (a) By means of a van de Graaff generator, protons are accelerated through a potential difference of $5 \times 10^6$ volts. The proton beam then passes through a thin silver foil. The atomic number of silver is 47, and you may assume that a silver nucleus is so massive compared with the proton that its motion may be neglected. What is the closest possible distance of approach, of any proton, to a silver nucleus? What will be the strength of the electric field acting on the proton at that position?

   (b) Some 40 years after Purcell wrote the above question, the Large Hadron Collider at CERN is accelerating beams of protons through a potential difference of $7 \times 10^{12}$ volts. Two beams of such protons are then collided head-on. What is the closest possible distance of approach of any pair of protons?

4. A thin disk, radius $a = 3$ cm, has a circular hole of radius $b = 1$ cm in the middle. There is a uniform surface charge of $\sigma = -4 \text{ esu/cm}^2$ on the disk.

   (a) What is the potential at the center of the hole? Answer in terms of $a$, $b$, and $\sigma$, then compute the value in statvolts. (Assume zero potential at infinite distance.)

   (b) An electron, starting from rest at the center of the hole, moves out along the axis, experiencing no forces except repulsion by the charges on the disk. What velocity does it ultimately attain? (Electron mass = $9 \times 10^{-28}$ gram.)
5. One of two nonconducting spherical shells of radius $a$ carries a charge $Q$ uniformly distributed over its surface, the other a charge $-Q$, also uniformly distributed. The spheres are brought together until they touch.

(a) What does the electric field look like, both outside and inside the shells?
(b) How much work is needed to move them far apart?

6. Consider a charge distribution which has the constant density $\rho$ everywhere inside a cube of edge $b$ and is zero everywhere outside that cube. Letting the electric potential $\phi$ be zero at infinite distance from the cube of charge, denote by $\phi_0$ the potential at the center of the cube and $\phi_1$ the potential at a corner of the cube. Determine the ratio $\phi_0/\phi_1$. The answer can be found with very little calculation by combining a dimensional argument with superposition. (Think about the potential at the center of a cube with the same charge density and with twice the edge length.)

7. A flat nonconducting sheet lies in the $xy$ plane. The only charges in the system are on this sheet. In the half-space above the sheet, $z > 0$, the potential is $\phi = \phi_0 e^{-kz} \cos kx$, where $\phi_0$ and $k$ are constants.

(a) Verify that $\phi$ satisfies Laplace’s equation in the space above the sheet.
(b) What do the electric field lines look like?
(c) Describe the charge distribution on the sheet.

8. How long did this problem set take you to complete?