1. (a) Show that, if a battery of fixed emf $E$ and internal resistance $R_i$ is connected to a variable external resistance $R$, the maximum power is delivered to the external resistor when $R = R_i$.

(b) A resistor $R$ is to be connected across the terminals $A$, $B$, of the circuit below. For what value of $R$ will the power dissipated in the resistor be greatest? To answer this, construct the Thévenin equivalent circuit and then invoke the result of (a). How much power will be dissipated in $R$?

![Diagram](image)

2. (a) Consider the $RC$ circuit discussed in Section 4.11 of the textbook, in which a capacitor $C$, initially holding a charge $Q$, is discharged through a resistor $R$. Show that the total energy dissipated in the resistor agrees with the energy originally stored in the capacitor.

(b) Consider the $RC$ circuit discussed in Lecture 10, in which the capacitor is charged from $Q = 0$ by an emf $E$ through a resistor $R$. Show that the sum of the energy dissipated in the resistor and the energy stored in the capacitor at the end ($t = \infty$) equals the energy delivered by the emf.

3. Two graphite rods are of equal length. One is a cylinder of radius $a$. The other is conical, tapering linearly from radius $a$ at one end to radius $b$ at the other. Show that the end-to-end electrical resistance of the conical rod is $a/b$ times that of the cylindrical rod. *Hint:* Consider the rod made up of thin, disk-like slices, all in series.

4. The Large Hadron Collider, currently under construction at CERN, Geneva, will accelerate protons to 7 TeV ($7 \times 10^{12}$ electron-volts). The protons will circulate in a 27-km long circular loop, which straddles the Franco-Swiss border. Calculate the strength of the vertical magnetic field that keeps the protons in this circular orbit.
5. All networks can be drawn flat if we adopt a conventional way of representing a “crossing without touching” such as X. Suppose a cube has a resistor along each edge. At each corner the leads from three resistors are soldered together.

(a) Flatten this network out into a circuit diagram.
(b) Find the equivalent resistance between two nodes that represent diagonally opposite corners of the cube, in the case where all resistors have the same value $R_0$. For this you do not need to solve a number of simultaneous equations; instead use symmetry arguments.
(c) Now find the equivalent resistance between two nodes that correspond diagonally opposite corners of one face of the cube. Here again, consideration of symmetry will reduce the problem to a very simple one.

For both calculations, a sketch of the structure as a cube, rather than flattened out, will help you to spot the necessary symmetries in the currents.

6. Some important kinds of networks are infinite in extent. The figure shows a chain of series and parallel resistors stretching off endlessly to the right. The line at the bottom is the resistanceless return wire for all of them. This is sometimes called an attenuator chain, or a ladder network.

(a) Find the “input resistance,” that is, the equivalent resistance between terminals $A$ and $B$. Our interest in this problem mainly concerns the method of solution, which takes an odd twist and which can be used in other places in physics where we have an iteration of identical devices (even an infinite chain of lenses, in optics). The point is that the input resistance which we do not yet know—call it $R$—will not be changed by adding a new set of resistors to the front end of the chain to make it one unit longer. But now, adding this section, we see that this new input resistance is just $R_1$ in series with the parallel combination of $R_2$ and $R$. We get immediately an equation that can be solved for $R$.
(b) Show that, if voltage $V_0$ is applied at the input to such a chain, the voltage at successive nodes decreases in a geometric series.
(c) What ratio is required for the resistors to make the ladder an attenuator that halves the voltage at every step?
(d) Obviously a truly infinite ladder would not be practical. Can you suggest a way to terminate it after a few sections without introducing any error in its attenuation?

7. How long did this problem set take you to complete?