1. (Purcell 5.3) A beam of 9.5-MeV electrons (\(\gamma = 20\)) amounting as current to 0.05 microamperes is traveling through vacuum. The transverse dimensions of the beam are less than 1 mm, and there are no positive charges in or near it.

(a) In the lab frame, what is approximately the electric field strength 1 cm away from the beam, and what is the average distance between an electron and the next one ahead of it, measured parallel to the beam?

(b) Answer the same questions for the electron rest frame.

2. (Purcell 5.10) In the rest frame of a particle with charge \(q_1\), another particle with charge \(q_2\) is approaching, moving with velocity \(v\) not small compared with \(c\). If it continues to move in a straight line, it will pass a distance \(d\) form the position of the first particle. It is so massive that its displacement from the straight path during the encounter is small compared with \(d\). Likewise, the first particle is so massive that its displacement from its initial position while the other particle is nearby is also small compared with \(d\).

(a) Show that the increment in momentum acquired by each particle as a result of the encounter is perpendicular to \(v\) and in magnitude \(2q_1q_2/vd\). (Gauss’s law can be useful here.)

(b) Expressed in terms of the other quantities, how large must the masses of the particles be to justify our assumptions?

3. (Purcell 5.18) Consider a composite line charge consisting of several kinds of carriers, each with its own velocity. For one kind, \(k\), the linear density of charge measured in frame \(F\) is \(\lambda_k\) and the velocity is \(\beta_k c\) parallel to the line. The contribution of these carriers to the current in \(F\) is then \(I_k = \lambda_k \beta_k c\). How much do these \(k\)-type carriers contribute to the charge and current in a frame \(F'\) which is moving parallel to the line at velocity \(-\beta c\) with respect to \(F\)?

By following the steps we took in the transformations in Lecture #13, you should be able to show that

\[
\lambda'_k = \gamma \left(\lambda_k + \frac{\beta I_k}{c}\right) \quad I'_k = \gamma (I_k + \beta c \lambda_k)
\]

If each component of the linear charge density and current transforms in this way, then so must the total \(\lambda\) and \(I\):

\[
\lambda' = \gamma \left(\lambda + \frac{\beta I}{c}\right) \quad I' = \gamma (I + \beta c \lambda)
\]

You have now derived the Lorentz transformation to a parallel-moving frame for any line charge and current, whatever its composition.
4. (Purcell 7.2) A long straight wire is parallel to the \( y \) axis and passes through the point \( z = h \) on the \( z \) axis. A current \( I \) flows in this wire, returning by a remote conductor whose field we may neglect. Lying in the \( xy \) plane is a square loop with two of its sides, of length \( b \), parallel to the long wire. This loop slides with constant speed \( v \) in the \( \hat{x} \) direction. Find the magnitude of the electromotive force induced in the loop at the moment when the center of the loop crosses the \( y \) axis.

5. (Purcell 7.14) A metal crossbar of mass \( m \) slides without friction on two long parallel conducting rails a distance \( b \) apart. A resistor \( R \) is connected across the rails at one end; compared with \( R \), the resistance of bar and rails is negligible. There is a uniform field \( B \) perpendicular to the plane of the figure. At time \( t = 0 \) the crossbar is given a velocity \( v_0 \) toward the right. What happens then?

(a) Does the rod ever stop moving? If so when?
(b) How far does it go?
(c) How about conservation of energy?

6. (Purcell 7.18) A circular coil of wire, with \( N \) turns of radius \( a \), is located in the field of an electromagnet. The magnetic field is perpendicular to the coil, and its strength has the constant value \( B_0 \) over that area. The coil is connected by a pair of twisted leads to an external resistance. The total resistance of this closed circuit, including that of the coil itself, is \( R \). Suppose the electromagnet is turned off, its field dropping more or less rapidly to zero. The induced emf causes current \( I \) to flow around the circuit. Derive a formula for the total charge \( Q = \int I \, dt \) which passes through the resistor, and explain why it does not depend on the rapidity with which the field drops to zero.

7. (Purcell 7.22) A thin ring of radius \( a \) carries a static charge \( q \). (NB: the ring is not a conductor; the charge cannot move on the ring.) This ring is in a magnetic field of strength \( B_0 \), parallel to the ring’s axis, and is supported so that it is free to rotate about the axis. If the field is switched off, how much angular momentum will be added to the ring? Supposing the mass of the ring to be \( m \), show that the ring, if initially at rest, will acquire an angular velocity \( \omega = qB_0/2mc \). Notice that the result depends only on the initial and final values of the field strength, and not the rapidity of change.

8. In Lecture 14, we saw a hand-cranked AC voltage generator using Earth’s magnetic field. The spinning coil was circular, with a radius of 15 cm, and had 60 turns. The coil was oriented so that it was perpendicular to Earth’s magnetic field. When the crank was turned two revolutions per second, a sinusoidal voltage signal of 4.5 mV peak-to-peak was recorded. What is the strength of Earth’s magnetic field?

9. How long did this problem set take you to complete?