1. (Purcell 7.21) The figure shows a solenoid of radius $a_1$ and length $b_1$ located inside a longer solenoid of radius $a_2$ and length $b_2$. The total number of turns is $N_1$ on the inner coil, $N_2$ on the outer. Work out a formula for the mutual inductance $M$. (Assume $b_2$ is very large.)

2. A superconducting solenoid in the ATLAS detector (http://atlas.ch/s_magnet.html) is 2.4 m in diameter and 5.3 m long. The design field is 2 tesla. Estimate roughly the energy stored in the field of this coil, in joules. (If the solenoid “quenches,” i.e., loses superconductivity during operation, the stored energy must be dissipated by redirecting the current into a massive resistor; otherwise the coil would be destroyed by the heat.)

3. (Purcell 8.4) In the resonant circuit of the figure the dissipative element is a resistor $R'$ connected in parallel, rather than in series, with the $LC$ combination.

(a) Work out the differential equation that describes the time evolution of the voltage $V$ between the top and bottom junctions.
(b) Find the conditions that separates weak, strong, and critical damping.
(c) If a series $RLC$ and a parallel $R'LC$ circuits have the same $L$, $C$, and $Q$ (that is, the rate of energy loss per oscillatory cycle; see the last paragraph of page 300 of the textbook), how must $R'$ be related to $R$?
4. (Purcell 8.7) A resonant cavity of the form illustrated in the figure is an essential part of many microwave oscillators. It can be regarded as a simple $LC$ circuit. The inductance is that of a toroid with one turn; this inductor is connected directly to parallel-plate capacitors. Find an expression for the resonant frequency of this circuit and show by a sketch the configuration of the magnetic and electric fields.

5. (Purcell 8.10) This circuit is driven with an AC voltage source $V(t) = V_0 \exp(i \omega t)$. Find the frequency that makes the voltage and the current provided by the voltage source in phase with each other. (This is equivalent to making the total impedance of the circuit purely real.)
6. (Purcell 8.13) Show that, if the condition $R_1 R_2 = L/C$ is satisfied by the components of the circuit below, the difference in voltage between points $A$ and $B$ will be zero at any frequency. How can you use this circuit as a method of measurement of an unknown inductance? (Such a circuit is called an AC bridge.)

![Circuit Diagram]

7. Although we defined the impedance of circuit elements (resistors, capacitors, and inductors) as they are driven by a sinusoidal voltage $V(t) = V_0 \exp(i\omega t)$, this does not always have to be the case. In fact one can extend the definition to include voltages in the form of $V(t) = V_0 \exp(st)$, where $s$ is a complex constant. The impedances for $R$, $L$, and $C$ then become

$$Z_R(s) = R, \quad Z_L(s) = sL, \quad Z_C(s) = \frac{1}{sC}. $$

(a) Consider a simple $RC$ circuit (a closed loop with just an $R$ and a $C$). In the absence of any voltage source, the current $I$ in this circuit must satisfy

$$(Z_R + Z_C)I = 0.$$  

Show that the only non-trivial solution to this equation is an exponentially-decaying current with the time constant $RC$.

(b) Apply the same idea to a weakly-damped $RLC$ circuit.

(c) What can you say about the physically allowed range of $s$?

8. How long did this problem set take you to complete?