1. Consider a horizontal mass-spring system discussed in Lecture #1. The mass, which is 1.5 kg, is displaced 10 cm to the left (negative x) and then released. Twenty oscillations are observed in 1 minute. Find
   (a) The spring constant.
   (b) The equation describing the oscillation, i.e. $x = x(t)$.
   (c) The total energy of the spring-mass system.

2. Consider a simple model of a diatomic H$_2$ molecule. Suppose that the atoms can move only along the axis connecting their centers. The force (in Newtons) each atom receives from the other atom is given as
   \[ F(r) = \frac{1.6 \times 10^{-81}}{r^8} - \frac{1.6 \times 10^{-65}}{r^6} \]
   where $r$ is the distance between the two atoms in meters.
   (a) Calculate the equilibrium separation of the atoms.
   (b) Sketch the shape of $F(r)$ and the potential energy $E(r)$ versus $r$.
   (c) Write down the linearized equation of motion for $r$. Assume that the center of mass of the molecule is stationary.
   (d) What is the condition the amplitude of oscillation must satisfy for the linearization in (c) to remain valid?
   (e) What is the resonant frequency of the molecule?
   (f) Light of this frequency would be of what wavelength? Where is this in the light spectrum (X-rays? Blue?)

3. Consider the weakly-damped LCR oscillator discussed in Lecture #2.
   (a) Calculate the energy $E_C$ stored in the capacitor and the energy $E_L$ stored in the inductor as functions of time. Simplify the answer assuming $\gamma << \omega$. Show that the sum of $E_C$ and $E_L$ decays exponentially.
   (b) Calculate the power $P$ dissipated in the resistor as a function of time. Use the same assumption (small $\gamma$) to simplify the calculation. Show that
   \[ P = -\frac{d}{dt}(E_C + E_L), \]
   i.e., the decrease of the total energy agrees with the power dissipation.

4. A capacitor of 5 $\mu$F charged to 1 kV is discharged through an inductor of 2 $\mu$H. The total resistance in the circuit is 5 m$\Omega$.
   (a) Is this a weakly damped LCR circuit?
   (b) Find the time by which one-half the initial energy stored in the capacitor has been dissipated. The time is measured from the instant when discharge is started.
5. Consider the critically-damped LCR oscillator discussed in Lecture #2.
   (a) Show that \( x(t) = t e^{-\gamma t} \) satisfies the equation.
   (b) Find a linear combination of the two solutions that satisfies the initial condition \( q(0) = q_0 \) and \( I(0) = 0 \).
   (c) Prove that the general solution for this oscillator can cross the equilibrium \( q = 0 \) at most once.

6. Consider the forced oscillator discussed in Lecture #2.
   (a) The \( Q \) value characterizes not only the steady-state amplitude of the oscillation at the resonance frequency, but also how quickly the amplitude falls off when the frequency is slightly off from the resonance. Show that the amplitudes at angular frequencies
   \[
   \omega = \omega_0 \left( 1 \pm \frac{1}{2Q} \right)
   \]
   are smaller than the amplitude at the resonance approximately by a factor \( \sqrt{2} \). (This means that the width of the resonance peak is approximately \( \Gamma = \omega_0 / Q \) in angular frequency.) What are the phases of the oscillation at these frequencies?
   (b) Calculate the energy transmitted at the spring-mass connection. The power transmitted from the spring to the mass is given by the spring tension times the velocity of the mass. Integrate this over one cycle and divide the result by the period \( T \) to obtain the average energy flow. This should agree with both the average work done by the motor and the average energy loss due to the friction.

7. Find an example of a weakly-damped oscillator around your room. (You can swing a mouse holding it’s cord, for example.) Observe its oscillation and estimate its \( Q \) value.