Physics 15c Problem Set #8

Due on Friday, November 20, 4:00 PM

Problem 1
In Lecture #18, I showed you that

\[ E_T = \frac{4Z_1Z_2 e^{-ikzd}}{(Z_1 + Z_2)^2 e^{-i kd} - (Z_1 - Z_2)^2 e^{ikzd}} E_1. \]

From this, show that the transmittivity becomes 100% when the thickness \( d \) satisfies the condition for zero reflectivity.

Problem 2
Now we extend the problem of thin-film interference to allow the incoming light at an angle. The figure on the right shows a ray of light hitting the surface of a thin pane of glass, thickness \( d \), at an angle \( \theta_1 \). The wavelength in the air is \( \lambda \). Indices of refraction of the air and of the glass are 1 and \( n \), respectively. Find the condition for destructive interference which would minimize the reflection.

Problem 3
Electromagnetic waves are traveling from air into a thin layer of plasma and out into air. The thickness of the plasma is \( d \), and the plasma frequency is \( \omega_p \). We assume that the frequency \( \omega \) of the incoming waves is smaller than \( \omega_p \). We define the \( x \) and \( z \) axes as the direction of the polarization and propagation of the incoming waves, respectively.

(a) The dispersion relation was calculated in Lecture #17 to be \( c^2 k^2 = \omega^2 - \omega_p^2 \). The wavenumber for \( \omega < \omega_p \) is therefore imaginary, and can be written as \( k = \pm i\kappa \), where \( \kappa > 0 \). Write down the expression for the electric field \( E(z,t) \) inside the plasma. (There are two independent solutions, one that decreases with \( z \) and one that increases with \( z \).)

(b) Find the magnetic field \( B(z,t) \) inside the plasma. (You need to use one of the Maxwell’s equations.)
(c) Use the continuity of \( \mathbf{E} \) and \( \mathbf{H} = \mathbf{B}/\mu_0 \) at \( z = 0 \) and \( d \) to find the reflectivity and the transmittivity.

**Problem 4**
For 2-body interference, a pretty-good approximate solution was given as

\[
I = I_0 \cos^2 \frac{\pi d \sin \theta}{\lambda}.
\]

How does the number of node lines (directions along which the wave intensity is zero) \( N \) depend on the distance \( d \) and the wavelength \( \lambda \)? Draw a graph of \( N \) vs. \( d/\lambda \).

**Problem 5**

(a) I am driving home while listening to WBUR (90.9 MHz). I stop at a red light, in front of a tall building whose vertical wall reflects radio waves, and my radio goes silent. How far away is the antenna of my car radio from the building? (Feel free to make simplifying assumptions about the shape and angle of the building, street, etc.) Would it help if I moved my car forward a few feet?

(b) If something similar happens to your cellular phone, how much movement is needed to get out of the dead zone? (In reality, cell phone dead zones are mostly shadows of the buildings, so this probably won’t help.)